PHYS5150 — PLASMA PHYSICS

LECTURE 4 - PLASMA PROPERTIES: PLASMA FREQUENCY

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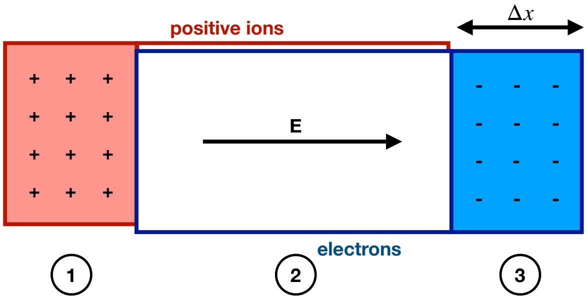
Plasma properties: Plasma Frequency and Plasma Criteria

1 PLASMA FREQUENCY

Because of charge neutrality a polarization E field will arise from charge imbalances, which eventually will reestablish the neutrality. It is a reasonable assumption that the magnitude of the charge imbalance is proportional to the charge displacement, suggesting that we can describe the response of the plasma to a charge imbalance by a restoring force given by *Hooke's law*, i.e $F = -k\Delta x$.

Let us now consider a slab of electrons of density n_0 and a background of immobile ions of the same density. Now we displace the slab by Δx:



The field strength is given by *Gauss' law*

$$\nabla \mathbf{E}(r) = \frac{n_0 e}{\epsilon_0}$$

$$E = \frac{n_0 e}{\epsilon_0} \Delta x,$$

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which gives us the equation of motion for the electrons

$$F = m_e \frac{\mathrm{d}^2}{\mathrm{d}t^2}\Delta x = -eE = -\frac{n_0 e^2}{\epsilon_0}\Delta x,$$

$$0 = \frac{\mathrm{d}^2}{\mathrm{d}t^2}\Delta x + \frac{n_0 e^2}{\epsilon_0 m_e}\Delta x.$$

The resulting equation describes an *harmonic oscillator* $\ddot{x} + \omega_0^2 x = 0$, where

$$\boxed{\omega_0 = \sqrt{\frac{n_0 e^2}{\epsilon_0 m_e}}} \qquad (1)$$

is the *plasma frequency*. The resulting plasma oscillations, discovered by *Irving Langmuir* and *Lewi Tonks* in 1929, are called *Langmuir* waves. Note that ω_0 does not depend on the wavelength λ implying that the corresponding phase velocity is proportional to λ and the group velocity vanishes, i.e. there is no charge transport.

Note also that the product of the Debye length and the plasma frequency

$$\lambda_D^2 \omega_0^2 = \frac{\epsilon_0 \mathrm{k_B} T_e}{n_0 e^2}\frac{n_0 e^2}{\epsilon_0 m_e} = \frac{\mathrm{k_B} T_e}{m_e} = c_s^2$$

is the thermal velocity.

2 EXAMPLE: SPONTANEOUS CHARGE FLUCTUATIONS

Let's calculate the radius r_m of a sphere that could be spontaneously depleted of all electrons due to thermal fluctuations. In this case all electrons previously within the sphere

$$N_{e^-} = \frac{4}{3}\pi r_m^3 n$$

would move through the outer boundary of the sphere, resulting in a total ion charge within the sphere of $Q = eN_{e^-}$. The corresponding electric field E_r can be found from Gauss' law

$$Q = \epsilon_0 \oint \mathbf{E}\,\mathrm{d}\mathbf{A} = \epsilon_0 E_r \oint \mathrm{d}\mathbf{A} = \epsilon_0 E_r 4\pi r_m^2,$$

$$E_r = \frac{Q}{4\pi\epsilon_0 r_m^2}.$$

Because the energy density of an electric field is $\frac{1}{2}\epsilon_0 E^2$, we can can calculate the total energy resulting from breaking the charge neutrality:

$$W = \int_0^{r_m} \frac{1}{2}\epsilon_0 E^2 4\pi\epsilon_0 r^2 \, \mathrm{d}r = \pi r_m^5 \frac{2n^2e^2}{45\epsilon_0}.$$

We wish W to be equal to thermal energy of the electrons

$$W_{th} = \frac{3}{2} n \mathrm{k_B} T \frac{4}{3}\pi r_m^3 = W,$$

which implies that

$$r_m^2 = 45\frac{\epsilon_0 \mathrm{k_B} T}{ne^2} = 45\lambda_D^2,$$

or

$$\boxed{r_m \approx 7\lambda_D}$$

3 CRITERIA FOR A PLASMA

We are now prepared to discuss when we will observe plasma effects:

DEBYE SHIELDING: For Debye shielding to happen the number of electrons within the Debye sphere must be large, i.e.

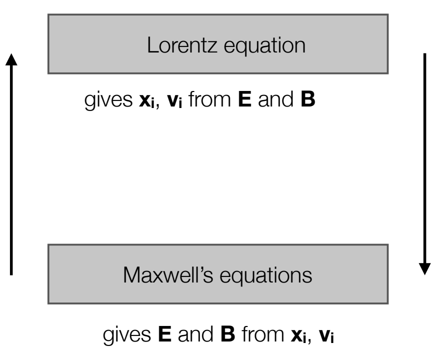
$$N_D = \frac{4}{3}\pi\lambda_D^3 n_e \gg 1.$$

PLASMA WAVES CAN PROPAGATE: For a wave to propagate the number of collisions during the oscillation time scale $1/\omega_0$ must be small.

THE DIMENSION OF THE SYSTEM MUST BE MUCH LARGER THAN λ_D

4 HOW TO SOLVE A PLASMA PROBLEM?

Recall from our first lecture that to solve a plasma problem means to find a good approximation for the system of about 10^{23} coupled equations:



So what can we do here? The general approach is to average over sub-groups of particles and to derive equations of motion for the resulting distributions:

VLASOW: For each specie σ we average over the particles at x with v ($\langle\ \rangle$), which gives us distributions $f_\sigma(\mathbf{x}, \mathbf{v}, t)$.

TWO FLUIDS APPROACH: For each specie σ we average over the particles at x ($\langle\ \rangle_v$), which gives us distributions for the density $n_\sigma(\mathbf{x}, t)$, mean velocity $\mathbf{u}(\mathbf{x}, t)$, and the pressure $P_\sigma(\mathbf{x}, t)$ (relative to the mean velocity).

MAGNETOHYDRODYNAMICS (MHD): We average over all species at x ($\langle\ \rangle_\sigma$), which gives us the center of mass density $\rho(\mathbf{x}, t)$, the center of mass velocity $\mathbf{u}(\mathbf{x}, t)$, and the pressure $P(\mathbf{x}, t)$ (relative to $\mathbf{u}(\mathbf{x}, t)$).