PHYS5150 — PLASMA PHYSICS

LECTURE 24 - ELECTROMAGNETIC WAVES IN UNMAGNETIZED PLASMAS

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1 EM WAVES IN MAGNETIZED PLASMAS

1.1 *Ordinary waves:* $\mathbf{k}\perp\mathbf{B}_0$ *and* $\delta\mathbf{E}\|\delta\mathbf{B}_0$

The geometry of an *ordinary wave* is such that

$$\delta\mathbf{B} = [0,0,B_z]$$
$$\delta\mathbf{E} = [0,0,E_z]$$
$$\mathbf{k} = [k_x,0,0].$$

Using Maxwell's equations similarly as in the unmagnetized case we get

$$\left(\omega^2 - k^{2c^2}\right) E_z = -\frac{i\omega}{\epsilon_0}\delta j_z = +\frac{i\omega n_0}{\epsilon_0} e\delta v_z.$$

The fluid's momentum equation gives us

$$v_z = -\frac{ie}{\omega m_e} E_z,$$

which results in the same dispersion relation as for the unmagnetized case

$$\boxed{\omega^2 = \omega_p^2 + k^2c^2.}$$

This finding explains why such waves are called *ordinary*.

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1.2 *Extraordinary waves:* $\mathbf{k}\perp\mathbf{B}_0$ *and* $\delta\mathbf{E}\perp\delta\mathbf{B}_0$

Extraordinary waves turn out to be polarized. Therefore we must include

$$\delta\mathbf{B} = [0,0,B_z]$$

$$\delta\mathbf{E} = [E_x,E_y,0]$$

$$\delta\mathbf{v} = [v_x,v_y,0]$$

$$\mathbf{k} = [k_x,0,0].$$

We start with the momentum equation

$$-i\omega m_e\delta\mathbf{v} = -e\left[\delta\mathbf{E}+\delta\mathbf{v}\times\mathbf{B}_0\right]$$

The x and y components are

$$\delta v_x = -\frac{ie}{m_e\omega}\left(E_x+\delta v_y B_0\right) \tag{1}$$

$$\delta v_y = -\frac{ie}{m_e\omega}\left(E_y-\delta v_x B_0\right) \tag{2}$$

We insert Eq.(2) into (1)

$$\delta v_x = -\frac{ie}{m_e\omega}\left(E_x-\frac{ie}{m_e\omega}\left(E_y-\delta v_x B_0\right)B_0\right)$$

and eventually find that

$$\delta v_x\left(1-\frac{\Omega_e^2}{\omega^2}\right) = -\frac{ie}{m_e\omega}\delta E_x-\frac{e\Omega_e}{m_e\omega^2}\delta E_y,$$

where $\Omega_e = \omega_{ce} = eB_0/m_e$ is the cyclotron frequency of the electrons. By the same method we also obtain the corresponding expression for δv_y:

$$\delta v_x\left(1-\frac{\Omega_e^2}{\omega^2}\right) = \frac{e}{m_e\omega}\left(-i\delta E_x-\frac{\Omega_e}{\omega}\delta E_y\right) \tag{3}$$

$$\delta v_y\left(1-\frac{\Omega_e^2}{\omega^2}\right) = \frac{e}{m_e\omega}\left(-i\delta E_y+\frac{\Omega_e}{\omega}\delta E_x\right) \tag{4}$$