PHYS5150 — PLASMA PHYSICS

LECTURE 24 - ELECTROMAGNETIC WAVES IN UNMAGNETIZED PLASMAS

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1 ELECTROMAGNETIC WAVES IN UNMAGNETIZED PLASMAS

We first study waves with $\mathbf{B} = 0$. We expect the basic characteristic of EM waves, i.e. $\mathbf{k}\perp\mathbf{B}\perp\mathbf{E}$.

Faraday's law:

$$\nabla \times \delta\mathbf{E} = -\frac{\partial \delta\mathbf{B}}{\partial t}$$

Ampere's law:

$$\nabla \times \delta\mathbf{B} = \mu\delta\mathbf{j} + \frac{1}{c^2}\frac{\partial \delta\mathbf{E}}{\partial t} = \frac{1}{c^2}\left[\underbrace{\frac{1}{\epsilon_0}\delta\mathbf{j}}_{\text{plasma effects}} - i\omega\delta\mathbf{E}\right]$$

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Take the curl of Faraday's law:

$$\nabla \times (\nabla \times \delta \mathbf{E}) = -\frac{\partial \nabla \times \delta \mathbf{B}}{\partial t}$$

$$\nabla(\nabla \cdot \delta \mathbf{E}) - \nabla^2 \mathbf{E} = i\omega \underbrace{(\nabla \times \delta \mathbf{B})}_{\text{use Ampere}}$$

$$-\mathbf{k} \underbrace{(\mathbf{k} \cdot \delta \mathbf{E})}_{=0} + k^2 \delta \mathbf{E} = \frac{i\omega}{c^2 \epsilon_0} \delta \mathbf{j} + \frac{\omega^2}{c^2} \delta \mathbf{E}$$

$$\left(\omega^2 - k^{2c^2}\right) \delta \mathbf{E} = -\frac{i\omega}{\epsilon_0} \delta \mathbf{j} = -\frac{i\omega n_0}{\epsilon_0} \sum_s q_s \delta \mathbf{v}_s$$

For the case of no plasma, i.e. $n_0 = 0$ we get the dispersion relation for a standard EM wave:

$$\boxed{\omega = ck = \frac{2\pi c}{\lambda}},$$

With plasma, we use the momentum equation to get a relation for $\delta \mathbf{v}$

$$n_0 m_e \frac{\partial \delta \mathbf{v}}{\partial t} = -n_0 e \delta \mathbf{E}$$

$$-i\omega \delta \mathbf{v} = -\frac{e}{m_e} \delta \mathbf{E}$$

$$\delta \mathbf{v} = -\frac{ie}{\omega m_e} \delta \mathbf{E},$$

and find that

$$\left(\omega^2 - k^{2c^2}\right) \delta \mathbf{E} = -\frac{i\omega n_0}{\epsilon_0} \frac{ie^2}{\omega m_e} \delta \mathbf{E} = \underbrace{\frac{n_0 e^2}{\epsilon_0 m_e}}_{\omega_p^2} \delta \mathbf{E}.$$

From this follows the dispersion relation for EM waves in an unmagnetized plasma

$$\boxed{\omega^2 = \omega_p^2 + k^2 c^2.}$$

For such a wave the phase velocity is

$$v_{ph}^2 = \frac{\omega^2}{k^2} = c^2 + \frac{\omega_p^2}{k^2} > c^2$$

always larger than the speed of light, while the group velocity (of course) is

$$v_{gr}^2 = \frac{\mathrm{d}\omega}{\mathrm{d}k} = \frac{k}{\omega} c^2 = \frac{c^2}{v_{ph}} < c$$

lower than the speed of light.

Now let's have a closer look at the *index of refraction*

$$\boxed{n = \frac{c}{v_{ph}}.}$$

For an EM wave propagation through an unmagnetized plasma we yield

$$n = \frac{c}{\sqrt{c^2 + \frac{\omega_p^2}{k^2}}} = \frac{c}{\omega} k = \sqrt{1 - \frac{\omega_p^2}{\omega}}.$$

This implies that for $\omega_p > \omega$ the index of refraction is imaginary, i.e. the wave is reflected.