PHYS5150 — PLASMA PHYSICS

LECTURE 18 - RECONNECTION

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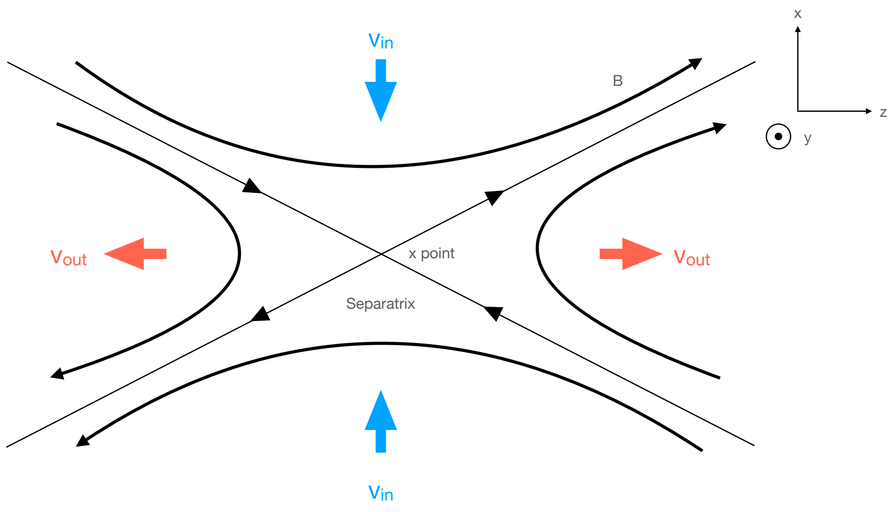
Spring 2023

1 RECONNECTION

Reconnection is the rearrangement of magnetic field lines entrained in a plasma. This happens by locally breaking the "frozen-in" flux condition, which we have investigated in lecture 15.

The basic reconnection configuration is looking like this:



The boundary between the ingoing and the outgoing plasma forms a separatrix. The central point of the separatrix is called the x-point. Reconnection is typically modeled with MHD with a non-zero resistivity at the x-point.

We now represent the 2D B-field lines as y-component of a vector potential $\mathbf{A}(\mathbf{r},t)$: $\mathbf{B} = \nabla \times \mathbf{A}$, $\mathbf{E} = \frac{\partial}{\partial t}\mathbf{A}$

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$$B_z = \frac{\partial}{\partial x} A_y$$

$$B_x = -\frac{\partial}{\partial z} A_y$$

For the chosen geometry, $B_z > 0$ at $z > 0$, implying that A_y has a local minimum at the x-point in cut along x and a local maximum in a cut along z. In other words, the x-point is a *saddle point* for A_y.

Now let's use *Ohm's law*

$$\mathbf{E} = -\mathbf{u} \times \mathbf{B} + \eta \mathbf{j},$$

where $\eta \neq 0$ near the x-line. Then,

$$E_y = u_x \frac{\partial B_z}{\partial x} - u_z \frac{\partial B_x}{\partial z} + \eta j_y$$

$$-\frac{\partial A_y}{\partial t} = u_x \frac{\partial A_y}{\partial x} + u_z \frac{\partial A_y}{\partial z} + \eta j_y$$

$$\underbrace{\frac{dA_y}{dt}}_{\text{moving with fluid}} = \underbrace{\frac{\partial A_y}{\partial t}}_{\text{in fixed frame}} + \mathbf{u} \cdot \nabla A_y = \underbrace{-\eta j_y}_{r}.$$

The current density at the x-line is negative, implying that

$$r = -\eta j_y \Big|_{\text{x-line}} > 0$$

and

$$\frac{dA_y}{dt}\Big|_{\text{x-line}} = E_y \Big|_{\text{x-line}} = r.$$

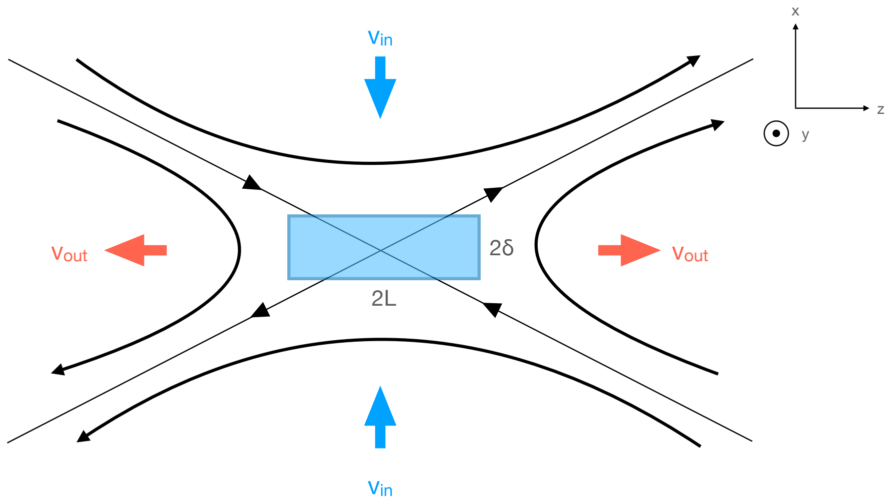
In other words, at the separatrix A_y is increasing at rate r. Since A_y increases with x this means that field lines originally in the flow region eventually become separatrixes. Another way of looking at this is to say that separatrixes become outflows.

Also note that the vector potential A_y is a straightforward measure of the *magnetic flux* in the x-z plane. A plot of A_y contours produces a 2-D field line map.

The change of $\frac{dA_y}{dt}|_{\text{x-line}}$ is a direct measure of how much magnetic flux is moved from the inflow to the outflow region. Thus, $r = -\eta j_y$ has the meaning of a *reconnection rate*.

1.1 *The Sweet-Parker model*

"How fast is the reconnection rate" is the key question, because this allows us to related theoretical predictions to observations. We can use the MHD equations to model the conditions inside and outside the x-point regions.



We start with continuity:

$$u_{in}L = u_{out}\delta \;\rightarrow\; \frac{u_{in}}{u_{out}} = \frac{\delta}{L}.$$

Next we use Ohm's law. Outside the layer, its y-component reads

$$E_y + u_{in}B_z = 0. \tag{1}$$

Inside the layer the flow is stagnated and thus, $u \approx 0$:

$$E_y = \eta j_y. \tag{2}$$

Faraday's law gives

$$\nabla \times \mathbf{E} = \underbrace{-\frac{\partial}{\partial t}\mathbf{B}}_{\text{(steady-state)}} = 0,$$

implying that E_z is in- and outside the layer the same. We assume that all B_z is annihilated, which implies for the pressure balance that

$$\frac{B_x}{2\mu_0} = \frac{\rho u_{out}^2}{2},$$

and thus

$$u_{out} = \frac{B_x}{\sqrt{\mu_0\rho}} = u_{alfven}.$$

We employ Ampere's law

$$\int B \cdot dl = \mu_0 i_{enclosed}$$

to find

$$B_z(4L) = \mu_0 j_y (2L)(2\delta),$$

hence

$$B_z = \mu_0 j_y \delta. \tag{3}$$

After combining Ohm's law (1) with Eq. (3)

$$u_{in} = \frac{E_y}{B_z} = \frac{\eta j_y}{\mu_0 j_y \delta} = \frac{\eta}{\mu_0 \delta}$$

or

$$\frac{u_{in}\mu_0\delta}{\eta} \equiv R_m = 1.$$

$R_m = vL_B\mu_0\sigma$

The inflow speed and the layer width adjust themself such that the magnetic Reynolds number (ratio of convection to diffusion) is unity. In other words, flux is annihilated as fast as plasma can be exhausted outsides.

Finally, the reconnection rate can be written as

$$r = \frac{\eta B_z}{\mu_0 \delta} = \frac{\eta B_z u_A}{\mu_0 L u_{in}}.$$

This is much faster than with now flow, but still not fast enough to match observations.